# Interactions and Strings in Higher Order Anisotropic and Inhomogeneous Superspaces and Isospaces

(summary of monograph)

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#### **Preface**

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This monograph gives a general geometric background of the theory of field interactions, strings and diffusion processes on spaces, superspaces and isospaces with higher order anisotropy and inhomogenity. The last fifteen years have been attempted a number of extensions of Finsler geometry with applications in theoretical and mathematical physics and biology which go far beyond the original scope. Our approach proceeds by developing the concept of higher order anisotropic superspace which unify the logical and mathematical aspects of modern Kaluza–Klein theories and generalized Lagrange and Finsler geometry and leads to modelling of physical processes on higher order fiber bundles provided with nonlinear and distingushed connections and metric structures. The view adopted here is that a gen-

eral field theory should incorporate all possible anisotropic and stochastic manifestations of classical and quantum interactions and, in consequence, a corresponding modification of basic principles and mathematical methods in formulation of physical theories. This monograph can be also considered as a pedagogical survey on the mentioned subjects.

There are established three approaches for modeling field interactions The first one is to deal with a usual locally and spaces anisotropies. isotropic physical theory and to consider anisotropies as a consequence of the anisotropic structure of sources in field equations (for instance, a number of cosmological models are proposed in the framework of the Einstein theory with the energy-momentum generated by anisotropic matter, as a general reference see [165]). The second approach to anisotropies originates from the Finsler geometry [78,55,213,159] and its generalizations [17,18,19,160,161,13,29, 256, 255, 264, 272] with a general imbedding into Kaluza–Klein (super) gravity and string theories [269,270,260,265,266,267], and speculates a generic anisotropy of the space—time structure and of fundamental field of interactions. The Santilli's approach [217,219,220,218,221, 222,223,224] is more radical by proposing a generalization of Lie theory and introducing isofields, isodualities and related mathematical structures. Roughly speaking, by using corresponding partitions of the unit we can model possible metric anisotropies as in Finsler or generalized Lagrange geometry but the problem is also to take into account classes of anisotropies generated by nonlinear and distinguished connections.

In its present version this book addresses itself both to mathematicians and physicists, to researches and graduate students which are interested in geometrical aspects of fields theories, extended (super)gravity and (super)strings and supersymmetric diffusion. It presupposes a general background in the mentioned divisions of modern theoretical physics and assumes some familiarity with differential geometry, group theory, complex analysis and stochastic calculus.

The monograph is divided into three parts:

The first five Chapters cover the higher order anisotropic supersymmetric theories: Chapter 1 is devoted to the geometry of higher order anisotropic supersaces with an extension to supergravity models in Chapter 2. The supersymmetric nearly autoparallel maps of superbundles and higher order anisotropic conservation laws are considered in Chapter 3. Higher order anisotropic superstrings and anomalies are studied in Chapter 4. Chap-

ter 5 contains an introduction into the theory of supersymmetric locally anisotropic stochastic processes.

The next five Chapters are devoted to the (non supersymmetric) theory of higher order anisotropic interactions and stochastic processes. Chapter 6 concerns the spinor formulation of field theories with locally anisotropic interactions and Chapter 7 considers anisotropic gauge field and gauge gravity models. Defining nearly autoparallel maps as generalizations of conformal transforms we analyze the problem of formulation of conservation laws in higher order anisotropic spaces in Chapter 8. Nonlinear sigma models and strings in locally anisotropic backgrounds are studied in Chapter 9. Chapter 10 is devoted to the theory of stochastic differential equations for locally anisotropic diffusion processes.

The rest four Chapters presents a study on Santilli's locally anisotropic and inhomogeneous isogeometries, namely, an introduction into the theory of isobuncles and generalized isofinsler gravity. Chapter 11 is devoted to basic notations and definitions on Santilli and coauthors isotheory. We introduce the bundle isospaces in Chapter 12 where some necessary properties of Lie-Santilli isoalgebras and isogroups and corresponding isotopic extensions of manifolds are applied in order to define fiber isospaces and consider their such (being very important for modeling of isofield interactions) classes of principal isobundles and vector isobundles. In that Chapter there are also studied the isogeometry of nonlinear isoconnections in vector isobundles, the isotopic distinguishing of geometric objects, the isocurvatures and isotorsions of nonlinear and distinguished isoconnections and the structure equations and invariant conditions. The next Chapter 13 is devoted to the isotopic extensions of generalized Lagrange and Finsler geometries. In Chapter 14 the isofield equations of locally anisotropic and inhomogeneous interactions will be analyzed and an outlook and conclusions will be presented.

We have not attempted to give many details on previous knowledge of the subjects or complete list of references. Each Chapter contains a brief introduction, the first section reviews the basic results, original papers and monographs. If it is considered necessary, outlook and discussion are presented at the end of the Chapter.

We hope that the reader will not suffer too much from our insufficient mastery of the English language.

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#### Part I. Higher Order Anisotropic Supersymmetry

## Capter 1. **HA–Superspaces**

The differential supergeometry have been formulated with the aim of getting a geometric framework for the supersymmetric field theories (see the theory of graded manifolds [37,146,147,144], the theory of supermanifolds [290,203,27,127] and, for detailed considerations of geometric and topological aspects of supermanifolds and formulation of superanalysis, [63,49,157, 114,281,283]). In this Chapter we apply the supergeometric formalism for a study of a new class of (higher order anisotropic) superspaces.

The concept of local anisotropy is largely used in some divisions of theoretical and mathematical physics [282,119,122,163] (see also possible applications in physics and biology in [14,13]). The first models of locally anisotropic (la) spaces (la–spaces) have been proposed by P.Finsler [78] and E.Cartan [55] (early approaches and modern treatments of Finsler geometry and its extensions can be found, for instance, in [213,17,18,159]). In our works [256,255,258,259,260,264,272,279,276] we try to formulate the geometry of la–spaces in a manner as to include both variants of Finsler and Lagrange, in general supersymmetric, extensions and higher dimensional Kaluza–Klein (super)spaces as well to propose general principles and methods of construction of models of classical and quantum field interactions and stochastic processes on spaces with generic anisotropy.

We cite here the works [31,33] by A. Bejancu where a new viewpoint on differential geometry of supermanifolds is considered. The author introduced the nonlinear connection (N-connection) structure and developed a corresponding distinguished by N-connection supertensor covariant differential calculus in the frame of De Witt [290] approach to supermani-

folds in the framework of the geometry of superbundles with typical fibres parametrized by noncommutative coordinates. This was the first example of superspace with local anisotropy. In our turn we have given a general definition of locally anisotropic superspaces (la-superspaces) [260]. It should be noted here that in our supersymmetric generalizations we were inspired by the R. Miron, M. Anastasiei and Gh. Atanasiu works on the geometry of nonlinear connections in vector bundles and higher order Lagrange spaces [160,161,162]. In this Chapter we shall formulate the theory of higher order vector and tangent superbundles provided with nonlinear and distinguished connections and metric structures (a generalized model of la-superspaces). Such superbundles contain as particular cases the supersymmetric extensions and various higher order prolongations of Riemann, Finsler and Lagrange spaces. We shall use instead the terms distinguished superbundles, distinguished geometric objects and so on (geometrical constructions distinguished by a N-connection structure) theirs corresponding brief denotations (d-superbundles, d-objects and so on).

The Chapter is organized as follows: Section 1.1 contains a brief review on supermanifolds and superbundles and an introduction into the geometry of higher order distinguished vector superbundles. Section 1.2 deals with the geometry of nonlinear and linear distinguished connections in vector superbundles and distinguished vector superbundles. The geometry of the total space of distinguished vector superbundles is studied in section 1.3; distinguished connection and metric structures, their torsions, curvatures and structure equations are considered. Generalized Lagrange and Finsler superspaces there higher order prolongations are defined in section 1.4. Concluding remarks on Chapter 1 are contained in section 1.5.

#### Chapter 2. **HA–Supergravity**

In this Chapter we shall analyze three models of supergravity with higher order anisotropy. We shall begin our considerations with N-connection spaces in section 2.1. Such s-spaces are generalizations of flat s-spaces containing a nontrivial N-connection structure but with vanishing d-connection. We shall introduce locally adapted s-vielbeins and define s-fields and differential forms in N-connection s-spaces. Sections 2.2 and 2.3 are correspondingly devoted to gauge s-field and s-gravity theory in osculator s-bundles. In order to have the possibility to compare our model with usual N=1 ( one dimensional supersymmetric extensions; see, for instance,

286,170,215) supergravitational models we develop a supergavity theory on osculator s-bundle  $Osc^zM_{(M)}$  where the even part of s-manifold  $M_{(M)}$ has a local structure of Minkowski space with action of Poincare group. In this case we do not have problems connected with definition of spinors (Lorentz, Weyl or Maiorana type) for spaces of arbitrary dimensions and can solve Bianchi identities. As a matter of principle, by using our results on higher dimensional and locally anisotropic spaces, see [256,255] we can introduce distinguished spinor stuctures and develop variants of extended supergravity with general higher order anisotropy. This approach is based on global geometric constructions and allows us to avoid tedious variational calculations and define the basic field equations and conservation laws on s-spaces with local anisotropy. That why, in section 2.5, we introduce Einstein-Cartan equations on distinguished vector superbundles (locally parametrized by arbitrary both type commuting and anticommuting coordinates) in a geometric manner, in some line following the geometric background for Einstein realtivity, but in our case on dvs-bundels provided with arbitrary N-connection and distinguished torsion and metric structures. We can consider different models, for instance, with prescribed N-connection and torsions, to develop a Einstein-Cartan like theory, or to follow approaches from gauge gravity. In section 2.6 we propose a variant of gauge like higher anisotropic supergravity being a generalization to dvsbundles of models of locally anisotropic gauge gravity [258,259,272] (see also Chapter 7 in this monograph).

#### Chapter 3. Supersymmetric NA-Maps

The study of models of classical and quantum field interactions in higher dimension superspaces with, or not, local anisotropy is in order of the day. The development of this direction entails great difficulties because of problematical character of the possibility and manner of definition of conservation laws on la–spaces. It will be recalled that conservation laws of energy—momentum type are a consequence of existence of a global group of automorphisms of the fundamental Mikowski spaces. As a rule one considers the tangent space's automorphisms or symmetries conditioned by the existence of Killing vectors on curved (pseudo)Riemannian spaces. There are not any global or local automorphisms on generic la–spaces and in result of this fact, at first glance, there are a lot of substantial difficulties with formulation of conservation laws and, in general, of physical consistent field

theories with local anisotropy. R. Miron and M. Anastasiei investigated the nonzero divergence of the matter energy—momentum d—tensor, the source in Einstein equations on la—spaces, and considered an original approach to the geometry of time—dependent Lagrangians [12,160,161]. In a series of papers [249,276,263,273,274,278,279,252,275,277] we attempt to solve the problem of definition of energy-momentum values for locally isotropic and anisotropic gravitational and matter fields interactions and of conservation laws for basic physical values on spaces with local anisotropy in the framework of the theory of nearly geodesic and nearly autoparallel maps.

In this Chapter a necessary geometric background (the theory of nearly autoparallel maps, in brief na-maps, and tensor integral formalism) for formulation and investigation of conservation laws on higher order isotropic and anisotropic superspaces is developed. The class of na-maps contains as a particular case the conformal transforms and is characterized by corresponding invariant conditions for generalized Weyl tensors and Thomas parameters [227,230]. We can connect the na-map theory with the formalism of tensor integral and multitensors on distinguished vector superbundles. This approaches based on generalized conformal transforms of superspaces with or not different types of higher order anisotropy consist a new division of differential supergeometry with applications in modern theoretical and mathematical physics.

We note that in most cases proofs of our theorems are mechanical but rather tedious calculations similar to those presented in [230,252,263]. Some of them will be given in detail, the rest will be sketched. We shall omit splitting of formulas into even and odd components (see Chapter 8 on nearly autoparallel maps and conservation laws for higher order (non supersymmetric) anisotropic spaces).

Section 3.1 is devoted to the formulation of the theory of nearly autoparallel maps of dvs—bundles. The classification of na—maps and formulation of their invariant conditions are given in section 3.2. In section 3.3 we define the nearly autoparallel tensor—integral on locally anisotropic multispaces. The problem of formulation of conservation laws on spaces with local anisotropy is studied in section 3.4. Some conclusions are presented in section 3.5.

### Chapter 4. **HA–Superstrings**

The superstring theory holds the greatest promise as the unification the-

ory of all fundamental interactions. The superstring models contains a lot a characteristic features of Kaluza–Klein approaches, supersymmetry and supergravity, local field theory and dual models. We note that in the string theories the nonlocal one dimensional quantum objects (strings) mutually interacting by linking and separating together are considered as fundamental values. Perturbations of the quantized string are identified with quantum particles. Symmetry and conservation laws in the string and superstring theory can be considered as sweeping generalizations of gauge principles which consists the basis of quantum field models. The new physical concepts are formulated in the framework a "new" for physicists mathematical formalism of the algebraic geometry and topology [106].

The relationship between two dimensional  $\sigma$ -models and strings has been considered [153,80,53,229,7] in order to discuss the effective low energy field equations for the massless models of strings. Nonlinear  $\sigma$ -models makes up a class of quantum field systems for which the fields are also treated as coordinates of some manifolds. Interactions are introduced in a geometric manner and admit a lot of applications and generalizations in classical and quantum field and string theories. The geometric structure of nonlinear sigma models manifests the existence of topological nontrivial configuration, admits a geometric interpretation of conterterms and points to a substantial interrelation between extended supersymmetry and differential supergeometry. In connection to this a new approach based on nonlocal, in general, higher order anisotropic constructions seem to be emerging [254,269,261]. We consider the reader to be familiar with basic results from supergeometry (see, for instance, [63,147,290,203]), supergravity theories [86,215,288,286,287] and superstrings [115,289,139,140].

In this Chapter we shall present an introduction into the theory of higher order anisotropic superstrings being a natural generalization to locally anisotropic (la) backgrounds (we shall write in brief la-backgrounds, la-spaces and la-geometry) of the Polyakov's covariant functional-integral approach to string theory [193]. Our aim is to show that a corresponding low-energy string dynamics contains the motion equations for field equations on higher order anisotropic superspaces and to analyze the geometry of the perturbation theory of the locally anisotropic supersymmetric sigma models. We note that this Chapter is devoted to supersymmetric models of locally anisotropic superstrings (details on the so called bosonic higher order anisotropic strings are given in the Chapter 9).

The plan of presentation in the Chapter is as follows. Section 4.1 contains an introduction into the geometry of two dimensional higher order anisotropic sigma models and an locally anisotropic approach to heterotic strings. In section 4.2 the background field method for  $\sigma$ -models is generalized for a distinguished calculus locally adapted to the N-connection structure in higher order anisotropic superspaces. Section 4.3 is devoted to a study of Green-Schwartz action in distinguished vector superbundles. Fermi strings in higher order anisotropic spaces are considered in section 4.4. An example of one-loop and two-loop calculus for anomalies of locally anisotropic strings is presented in section 4.5. Conclusions are drawn in section 4.6.

## Chapter 5. Stochastics in LAS-Spaces

We shall describe the analytic results which combine the fermionic Brownian motion with stochastic integration in higher order anisotropic spaces. It will be shown that a wide class of stochastic differential equations in locally anisotropic superspaces have solutions. Such solutions will be than used to derive a Feynman–Kac formula for higher order anisotropic systems. We shall achieve this by introducing locally anisotropic superpaths parametrized by a commuting and an anticommuting time variable. The supersymmetric stochastic techniques employed in this Chapter was developed by A. Rogers in a series of works [206,207,205,209,210] (superpaths have been also considered in papers [103,82] and [198]). One of the main our purposes is to extend this formalism in order to formulate the theory of higher order anisotropic processes in distinguished vector superbundles [260,262,265,266,267,253,268]. Stochastic calculus for bosonic and fermionic Brownian paths will provide a geometric approach to Brownian motion in locally anisotropic superspaces.

Sections 5.1 and 5.2 of this Chapter contain correspondingly a brief introduction into the subject and a brief review of fermionic Brownian motion and path integration. Section 5.3 considers distinguished stochastic integrals in the presence of fermionic paths. Some results in calculus on a (1,1)-dimensional superspace and two supersymmetric formulae for superpaths are described in section 5.4 and than, in section 5.5, the theorem on the existence of unique solutions to a useful class of distinguished stochastic differential equations is proved and the distinguished supersymmetric

Feynman–Kac formula is established. Section 5.6 defines some higher order anisotropic manifolds which can be constructed from a vector bundle over a vector bundle provided with compatible nonlinear and distinguished connection and metric structures. In section 5.7 a geometric formulation of Brownian paths on higher order anisotropic manifolds is contained; these paths are used to give a Feynman–Kac formula for the Laplace–Beltrami operator for twisted differential forms. This formula is used to give a proof of the index theorem using supersymmetry of the higher order anisotropic superspaces in section 5.8 (we shall apply the methods developed in [4,82] and [209,210].

#### Part II. Higher Order Anisotropic Interactions

## Chapter 6. **HA–Spinors**

Some of fundamental problems in physics advocate the extension to locally anisotropic and higher order anisotropic backgrounds of physical theories [159,161,13,29,18,162,272,265,266,267]. In order to construct physical models on higher order anisotropic spaces it is necessary a corresponding generalization of the spinor theory. Spinor variables and interactions of spinor fields on Finsler spaces were used in a heuristic manner, for instance, in works [18,177], where the problem of a rigorous definition of la-spinors for la-spaces was not considered. Here we note that, in general, the nontrivial nonlinear connection and torsion structures and possible incompatibility of metric and connections makes the solution of the mentioned problem very sophisticate. The geometric definition of la-spinors and a detailed study of the relationship between Clifford, spinor and nonlinear and distinguished connections structures in vector bundles, generalized Lagrange and Finsler spaces are presented in refs. [256,255,264].

The purpose of the Chapter is to summarize and extend our investigations [256,255,264,272,260] on formulation of the theory of classical and quantum field interactions on higher order anisotropic spaces. We receive primary attention to the development of the necessary geometric framework: to propose an abstract spinor formalism and formulate the differential geometry of higher order anisotropic spaces. The next step is the investigation of higher order anisotropic interactions of fundamental fields on generic higher order anisotropic spaces (in brief we shall use instead of higher order anisotropic the abbreviation ha-, for instance, ha-spaces, ha-interactions

and ha-spinors).

In order to develop the higher order anisotropic spinor theory it will be convenient to extend the Penrose and Rindler abstract index formalism [180,181,182] (see also the Luehr and Rosenbaum index free methods [154]) proposed for spinors on locally isotropic spaces. We note that in order to formulate the locally anisotropic physics usually we have dimensions d > 4 for the fundamental, in general higher order anisotropic space—time and to take into account physical effects of the nonlinear connection structure. In this case the 2-spinor calculus does not play a preferential role.

Section 6.1 of this Chapter contains an introduction into the geometry of higher order anisotropic spaces, the distinguishing of geometric objects by N-connection structures in such spaces is analyzed, explicit formulas for coefficients of torsions and curvatures of N- and d-connections are presented and the field equations for gravitational interactions with higher order anisotropy are formulated. The distinguished Clifford algebras are introduced in section 6.2 and higher order anisotropic Clifford bundles are defined in section 6.3. We present a study of almost complex structure for the case of locally anisotropic spaces modeled in the framework of the almost Hermitian model of generalized Lagrange spaces in section 6.4. The d-spinor techniques is analyzed in section 6.5 and the differential geometry of higher order anisotropic spinors is formulated in section 6.6. The section 6.7 is devoted to geometric aspects of the theory of field interactions with higher order anisotropy (the d-tensor and d-spinor form of basic field equations for gravitational, gauge and d-spinor fields are introduced). Finally, an outlook and conclusions on ha-spinors are given in section 6.8.

## Chapter 7. Gauge and Gravitational Ha–Fields

Despite the charm and success of general relativity there are some fundamental problems still unsolved in the framework of this theory. Here we point out the undetermined status of singularities, the problem of formulation of conservation laws in curved spaces, and the unrenormalizability of quantum field interactions. To overcome these defects a number of authors (see, for example, Refs. [240,285,194,3]) tended to reconsider and reformulate gravitational theories as a gauge model similar to the theories of weak, electromagnetic, and strong forces. But, in spite of theoretical arguments and the attractive appearance of different proposed models of gauge gravity,

the possibility and manner of interpretation of gravity as a kind of gauge interaction remain unclear.

The work of Popov and Daikhin [195,196] is distinguished among other gauge approaches to gravity. Popov and Dikhin did not advance a gauge extension, or modification, of general relativity; they obtained an equivalent reformulation (such as well-known tetrad or spinor variants) of the Einstein equations as Yang-Mills equations for correspondingly induced Cartan connections [40] in the affine frame bundle on the pseudo-Riemannian space time. This result was used in solving some specific problems in mathematical physics, for example, for formulation of a twistor-gauge interpretation of gravity and of nearly autoparallel conservation laws on curved spaces [246,250,252,249,233]. It has also an important conceptual role. On one hand, it points to a possible unified treatment of gauge and gravitational fields in the language of linear connections in corresponding vector bundles. On the other, it emphasize that the types of fundamental interactions mentioned essentially differ one from another, even if we admit for both of them a common gauge like formalism, because if to Yang-Mills fields one associates semisimple gauge groups, to gauge treatments of Einstein gravitational fields one has to introduce into consideration nonsemisimple gauge groups.

Recent developments in theoretical physics suggest the idea that a more adequate description of radiational, statistical, and relativistic optic effects in classical and quantum gravity requires extensions of the geometric background of theories [282,163,13,14,15,28,29,118,119,120,122,212,235,236,238,280,41] by introducing into consideration spaces with local anisotropy and formulating corresponding variants of Lagrange and Finsler gravity and theirs extensions to higher order anisotropic spaces [295,162,266,267,268].

The aim of this Chapter is twofold. The first objective is to present our results [272,258,259] on formulation of geometrical approach to interactions of Yang-Mills fields on spaces with higher order anisotropy in the framework of the theory of linear connections in vector bundles (with semisimple structural groups) on ha-spaces. The second objective is to extend the geometrical formalism in a manner including theories with nonsemisimple groups which permit a unique fiber bundle treatment for both locally anisotropic Yang-Mills field and gravitational interactions. In general lines, we shall follow the ideas and geometric methods proposed in refs. [240,195,196,194,40] but we shall apply them in a form convenient for introducing into consider-

ation geometrical constructions [160,161] and physical theories on ha-spaces.

There is a number of works on gauge models of interactions on Finsler spaces and theirs extensions (see, for instance, [17,18,19,28,164,177]). One has introduced different variants of generalized gauge transforms, postulated corresponding Lagrangians for gravitational, gauge and matter field interactions and formulated variational calculus (here we note the approach developed by A. Bejancu [30,32,29]). The main problem of such models is the dependence of the basic equations on chosen definition of gauge "compensation" symmetries and on type of space and field interactions anisotropy. In order to avoid the ambiguities connected with particular characteristics of possible la-gauge theories we consider a "pure" geometric approach to gauge theories (on both locally isotropic and anisotropic spaces) in the framework of the theory of fiber bundles provided in general with different types of nonlinear and linear multiconnection and metric structures. This way, based on global geometric methods, holds also good for nonvariational, in the total spaces of bundles, gauge theories (in the case of gauge gravity based on Poincare or affine gauge groups); physical values and motion (field) equations have adequate geometric interpretation and do not depend on the type of local anisotropy of space-time background. It should be emphasized here that extensions for higher order anisotropic spaces which will be presented in this Chapter can be realized in a straightforward manner.

The presentation in the Chapter is organized as follows:

In section 7.1 we give a geometrical interpretation of gauge (Yang-Mills) fields on general ha-spaces. Section 7.2 contains a geometrical definition of anisotropic Yang-Mills equations; the variational proof of gauge field equations is considered in connection with the "pure" geometrical method of introducing field equations. In section 7.3 the ha–gravity is reformulated as a gauge theory for nonsemisimple groups. A model of nonlinear de Sitter gauge gravity with local anisotropy is formulated in section 7.4. We study gravitational gauge instantons with trivial local anisotropy in section 7.5. Some remarks are given in section 7.6.

### Chapter 8. Na–Maps and Conservation Laws

Theories of field interactions on locally anisotropic curved spaces form a new branch of modern theoretical and mathematical physics. They are used for modelling in a self-consistent manner physical processes in locally anisotropic, stochastic and turbulent media with beak radiational reaction and diffusion [161,13,14,282]. The first model of locally anisotropic space was proposed by P.Finsler [78] as a generalization of Riemannian geometry; here we also cite the fundamental contribution made by E. Cartan [55] and mention that in monographs [213,159,17,19,29] detailed bibliographies are contained. In this Chapter we follow R. Miron and M. Anastasiei [160,161] conventions and base our investigations on their general model of locally anisotropic (la) gravity (in brief we shall write la-gravity) on vector bundles, v-bundles, provided with nonlinear and distinguished connection and metric structures (we call a such type of v-bundle as a la-space if connections and metric are compatible).

The study of models of classical and quantum field interactions on laspaces is in order of the day. For instance, the problem of definition of spinors on la-spaces is already solved (see [256,275,264] and Chapter 6 and some models of locally anisotropic Yang-Mills and gauge like gravitational interactions are analyzed (see [272,263] and Chapter 7 and alternative approaches in [17,29,122,119]). The development of this direction entails great difficulties because of problematical character of the possibility and manner of definition of conservation laws on la-spaces. It will be recalled that, for instance, conservation laws of energy-momentum type are a consequence of existence of a global group of automorphisms of the fundamental Mikowski spaces (for (pseudo)Riemannian spaces the tangent space' automorphisms and particular cases when there are symmetries generated by existence of Killing vectors are considered). No global or local automorphisms exist on generic la-spaces and in result of this fact the formulation of la-conservation laws is sophisticate and full of ambiguities. R. Miron and M. Anastasiei firstly pointed out the nonzero divergence of the matter energy-momentum d-tensor, the source in Einstein equations on la-spaces, and considered an original approach to the geometry of time-dependent Lagrangians [12,160,161]. Nevertheless, the rigorous definition of energymomentum values for la-gravitational and matter fields and the form of conservation laws for such values have not been considered in present-day studies of the mentioned problem.

The aim of this Chapter is to develop a necessary geometric background (the theory of nearly autoparallel maps, in brief na-maps, and tensor integral formalism on la-multispaces) for formulation and a detailed investigation of conservation laws on locally isotropic and anisotropic curved spaces. We

shall summarize our results on formulation of na-maps for generalized affine spaces (GAM-spaces) [249,251,273], Einstein-Cartan and Einstein spaces [250,247,278] bundle spaces [250,247,278] and different classes of la-spaces [279,276,102,263] and present an extension of the na-map theory for superspaces. For simplicity we shall restrict our considerations only with the "first" order anisotropy (the basic results on higher order anisotropies a presented in a supersymmetric manner Chapter 3. Comparing the geometric constructions from both Chapters on na-map theory we assure ourselves that the developed methods hold good for all type of curved spaces (with or not torsion, locally isotropic or even with local anisotropy and being, or not, supesymmetric). In order to make the reader more familiar with na-maps and theirs applications and to point to some common features, as well to proper particularities of the supersymmetric and higher order constructions, we shall recirculate some basic definitions, theorems and proofs from Chapter 3.

The question of definition of tensor integration as the inverse operation of covariant derivation was posed and studied by A.Moór [167]. Tensorintegral and bitensor formalisms turned out to be very useful in solving certain problems connected with conservation laws in general relativity [100,247]. In order to extend tensor-integral constructions we have proposed [273,278] to take into consideration nearly autoparallel [249,247,250] and nearly geodesic [230] maps, ng-maps, which forms a subclass of local 1-1 maps of curved spaces with deformation of the connection and metric structures. A generalization of the Sinyukov's ng-theory for spaces with local anisotropy was proposed by considering maps with deformation of connection for Lagrange spaces (on Lagrange spaces see [136,160,161]) and generalized Lagrange spaces [263,279,276,275,101]. Tensor integration formalism for generalized Lagrange spaces was developed in [255,102,263]. One of the main purposes of this Chapter is to synthesize the results obtained in the mentioned works and to formulate them for a very general class of la-spaces. As the next step the la-gravity and analysis of la-conservation laws are considered.

We note that proofs of our theorems are mechanical, but, in most cases, they are rather tedious calculations similar to those presented in [230,252,263]. Some of them, on la-spaces, will be given in detail the rest, being similar, or consequences, will be only sketched or omitted.

Section 8.1 is devoted to the formulation of the theory of nearly au-

toparallel maps of la—spaces. The classification of na—maps and formulation of their invariant conditions are given in section 8.2. In section 8.3 we define the nearly autoparallel tensor—integral on locally anisotropic multispaces. The problem of formulation of conservation laws on spaces with local anisotropy is studied in section 8.4. We present a definition of conservation laws for la—gravitational fields on na—images of la—spaces in section 8.5. Finally, in this Chapter, section 8.6, we analyze the locally isotropic limit, to the Einstein gravity and it generalizations, of the na-conservation laws.

## Chapter 9. **HA–Strings**

The relationship between two dimensional  $\sigma$ -models and strings has been considered [153,80,53,229,7] in order to discuss the effective low energy field equations for the massless models of strings. In this Chapter we shall study some of the problems associated with the theory of higher order anisotropic strings being a natural generalization to higher order anisotropic backgrounds (we shall write in brief ha–backgrounds, ha–spaces and ha–geometry) of the Polyakov's covariant functional–integral approach to string theory [193]. Our aim is to show that a corresponding low–energy string dynamics contains the motion equations for field equations on ha-spaces; models of ha–gravity could be more adequate for investigation of quantum gravitational and Early Universe cosmology.

The plan of the Chapter is as follows. We begin, section 9.1, with a study of the nonlinear  $\sigma$ -model and ha–string propagation by developing the d–covariant method of ha–background field. Section 9.2 is devoted to problems of regularization and renormalization of the locally anisotropic  $\sigma$ -model and a corresponding analysis of one- and two–loop diagrams of this model. Scattering of ha-gravitons and duality are considered in section 9.3, and a summary and conclusions are drawn in section 9.4.

#### Chapter 10. Stochastic Processes on HA–Spaces

The purpose of investigations in this Chapter [253,262] is to extend the formalism of stochastic calculus to the case of spaces with higher order anisotropy (distinguished vector bundles with compatible nonlinear and distinguished connections and metric structures and generalized Lagrange and Finsler spaces). We shall examine nondegenerate diffusions on the mentioned spaces and theirs horizontal lifts.

Probability theorists, physicists, biologists and financiers are already familiar with classical and quantum statistical and geometric methods applied in various branches of science and economy [13,14,171,189,131,84,141]. We note that modeling of diffusion processes in nonhomogerneous media and formulation of nonlinear thermodynamics in physics, or of dynamics of evolution of species in biology, requires a more extended geometrical background than that used in the theory of stochastic differential equations and diffusion processes on Riemann, Lorentz manifolds [117,74,75,76] and in Rieman–Cartan–Weyl spaces [197,199].

Our aim is to formulate the theory of diffusion processes on spaces with local anisotropy. As a model of such spaces we choose vector bundles on space-times provided with nonlinear and distinguished connections and metric structures [160,161]. Transferring our considerations on tangent bundles we shall formulate the theory of stochastic differential equations on generalized Lagrange spaces which contain as particular cases Lagrange and Finsler spaces [213,17,18,29,159].

The plan of the presentation in the Chapter is as follow: We present a brief introduction into the theory of stochastic differential equations and diffusion processes on Euclidean spaces in section 10.1. In section 10.2 we give a brief summary of the geometry of higher order anisotropic spaces. Section 10.3 is dedicated to the formulation of the theory of stochastic differential equations in distinguished vector bundle spaces. This section also concerns the basic aspects of stochastic calculus in curved spaces. In section 10.4 the heat equations in bundle spaces are analyzed. The nondegenerate diffusion on spaces with higher order anisotropy is defined in section 10.5. We shall generalize in section 10.6 the results of section 10.4 to the case of heat equations for distinguished tensor fields in vector bundles with (or not) boundary conditions. Section 10.7 contains concluding remarks and a discussion of the obtained results.

## Part III. Isobundles and Generalized Isofinsler Gravity

## Chapter 11. Basic Notions on Isotopies

This Part is devoted to a generalization [271] of the geometry of Santilli's locally anisotropic and inhomogeneous isospaces [217,219,220,218,221,222,

223,224] to the geometry of vector isobundles provided with nonlinear and distinguished isoconnections and isometric structures. We present, apparently for the first time, the isotopies of Lagrange, Finsler and Kaluza–Klein spaces. We also continue the study of the interior, locally anisotropic and inhomogeneous gravitation by extending the isoriemannian space's constructions and presenting a geometric background for the theory of isofield interactions in generalized isolagrange and isofinsler spaces.

The main purpose of this Part is to formulate a synthesis of the Santilli isotheory and the approach on modeling locally anisotropic geometries and physical models on bundle spaces provided with nonlinear connection and distinguished connection and metric structures [160,161,295]. The isotopic variants of generalized Lagrange and Finsler geometry will be analyzed. Basic geometric constructions such as nonlinear isoconnections in vector isobundles, the isotopic curvatures and torsions of distinguished isoconnections and theirs structure equations and invariant values will be defined. A model of locally anisotropic and inhomogeneous gravitational isotheory will be constructed.

Our study of Santilli's isospaces and isogeometries over isofields will be treated via the isodifferential calculus according to their latest formulation [224] (we extend this calculus for isospaces provided with nonlinear isoconnection structure). We shall also use Kadeisvili's notion of isocontinuity [129,130] and the novel Santilli–Tsagas–Sourlas isodifferential topology [223,239,232].

After reviewing the basic elements for completeness as well as for notational convenience, we shall extend Santilli's foundations of the isosympletic geometry [223] to isobundles and related aspects (by applying, in an isotopic manner, the methods summarized in Miron and Anastasiei [160,161] and Yano and Ishihara [295] monographs). We shall apply our results on isotopies of Lagrange, Finsler and Kaluza–Klein geometries to further studies of the isogravitational theories (for isoriemannian spaces firstly considered by Santilli [223]) on vector isobundle provided with compatible nonlinear and distinguished isoconnections and isometric structures. Such isogeometrical models of isofield interaction isotheories are in general nonlinear, non-local and nonhamiltonian and contain a very large class of local anisotropies and inhomogeneities induced by four fundamental isostructures: the partition of unity, nonlinear isoconnection, distinguished isoconnections and isometric.

The novel geometric profile emerging from all the above studies is rather remarkable inasmuch as the first class of all isotopies herein considered (called Kadeisvili's Class I [129,130]) preserves the abstract axioms of conventional formulations, yet permits a clear broadening of their applicability, and actually result to be "directly universal" [223] for a number of possible well behaved nonlinear, nonlocal and nonhamiltonian systems. In turn, this permits a number of geometric unification such as that of all possible metrics (on isospaces with trivial nonlinear isoconnection structure) of a given dimension into Santilli's isoeuclidean metric, the unification of exterior and interior gravitational problems despite their sizable structural differences and other unification.

## Chapter 12. Isobundle Spaces

This chapter serves the twofold purpose of establishing of abstract index denotations and starting the geometric backgrounds of isotopic locally anisotropic extensions of the isoriemannian spaces which are used in the next chapters of the work.

## Chapter 13. The Isogeometry of Tangent Isobundles

The aim of this Chapter is to formulate some results in the isogeometry of tangent isobundle, t—isobundle,  $\widehat{TM}$  and to use them in order to develop the geometry of Finsler and Lagrange isospaces.

#### Chapter 14. Locally Anisotropic and Inhomogeneous Isogravity

The conventional Riemannian geometry can be generally assumed to be exactly valid for the exterior gravitational problem in vacuum where bodies can be well approximated as being massive points, thus implying the validity of conventional and calculus.

On the contrary, there have been serious doubt dating back to E. Cartan on the same exact validity of the Riemannian geometry for interior gravitational problem because the latter imply internal effects which are arbitrary nonlinear in the velocities and other variables, nonlocal integral and of general non–(first)–order Lagrangian type.

Santilli [221,222,224,223] constructed his isoriemannian geometry and

proposed the related isogravitation theory precisely to resolve the latter shortcoming. In fact, the isometric acquires an arbitrary functional; dependence thus being able to represent directly the locally anisotropic and inhomogeneous character of interior gravitational problems.

A remarkable aspect of the latter advances is that they were achieved by preserving the abstract geometric axioms of the exterior gravitation. In fact, exterior and interior gravitation are unified in the above geometric approach and are merely differentiated by the selected unit, the trivial value I = diag(1, 1, 1, 1) yielding the conventional gravitation in vacuum while more general realization of the unit yield interior conditions under the same abstract axioms (see ref. [129,130] for an independent study).

A number of applications of the isogeometries for interior problems have already been identified, such as (see ref. [224] for an outline): the representation of the local variation of the speed of light within physical media such as atmospheres or chromospheres; the representation of the large difference between cosmological redshift between certain quasars and their associated galaxies when physically connected according to spectroscopic evidence; the initiation of the study of the origin of the gravitation via its identification with the field originating the mass of elementary constituents.

As we have shown [269,261,254,270] the low energy limits of string and superstring theories give also rise to models of (super)field interactions with locally anisotropic and even higher order anisotropic interactions. The N-connection field can be treated as a corresponding nonlinear gauge field managing the dynamics of "step by step" splitting (reduction) of higher dimensional spaces to lower dimensional ones. Such (super)string induced (super)gravitational models have a generic local anisotropy and, in consequence, a more sophisticate form of field equations and conservation laws and of corresponding theirs stochastic and quantum modifications. Perhaps similar considerations are in right for isotopic versions of sting theories. That it is why we are interested in a study of models of isogravity with nonvanishing nonlinear isoconnection, distinguished isotorsion and, in general, non-isometric fields.

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